

A Short Course in Estimating Uncertainty for Measurement with the Electro-Limit Indicator

Why Calculate Measurement Uncertainty?

Estimates of measurement uncertainty are now an essential component of the metrology business. Anyone that intends to use precision measurement data needs to know the quality of that data. Calibration customers will look for the uncertainty estimate published on a scope of accreditation in order to determine if a supplier's calibration will support their desired uncertainty. Accreditation registrars require that accredited labs supply uncertainty estimates for any measurement they hope to post on their scope of accreditation. Manufacturers need to know if the tolerance uncertainty ratio for the measurement of a measured part is high enough for the measurement to be trusted. Users of calibrated gauges look for the uncertainty values reported on certificates of calibration in order to determine if a reference gauge block or ring gage can be used as a master for a needed measurement. The user of a Pratt & Whitney Electro-Limit Indicator or Electro-Limit Comparator may need to calculate estimates of measurement uncertainty for all those same reasons and more. The following scenarios are just a few examples of those situations where the user of an Electro-Limit Indicator might need to calculate an estimate of measurement uncertainty: The Electro-Limit Indicator user may seek accreditation for measurements of gage blocks made using the Electro-Limit Indicator – the user needs to generate an uncertainty value to show an auditor. The user of the Electro-Limit Indicator may need to determine if in-house measurement of a first piece part can be made using the Electro-Limit Indicator – the user needs to compare an uncertainty value to requisite tolerances. The organization that relies on the Electro-Limit Indicator for calibration of pins may need to generate certificates of calibration bearing the estimate of measurement uncertainty to communicate the quality of their pin calibration with customers.

The Process

Regardless of the intended use of the uncertainty value, the person responsible for calculating the estimate of measurement uncertainty will likely follow the same three essential steps: Determine sources of uncertainty, Estimate the magnitude of the impact of each source, Combine the values by Root-Sum-Squared arithmetic.

Determine Sources of Uncertainty

The first step of calculating an estimate of measurement uncertainty is to generate a comprehensive list of the sources of uncertainty that will affect the measurement in question. Some of those sources of uncertainty will be sources that are common to all measurement – likely repeatability, datum stability, and instrument resolution among others. Some will be required by standards like ISO 17025. Other sources of uncertainty may be very unique to the measurement in question either because the unit under test is unique, the measurement environment is unique, or the contact geometry is unique. It is also very important to understand that, whichever measurement is being considered, the estimate of measurement uncertainty is applicable only to the measurement that it is calculated for. For example, the uncertainty of measurement for measurement of a 1-inch gage block using the Electro-Limit

Indicator will not be the same as the uncertainty of measurement for measurement of a 1-inch gage pin using the Electro-Limit Indicator; the contact geometry will differ. Furthermore, the uncertainty of measurement for measurement of a 1-inch gage block using the Electro-Limit Indicator and NIST calibrated masters will not be the same as the uncertainty of measurement for measurement of a 1-inch gage block using the Electro-Limit Indicator and masters calibrated by a local calibration laboratory; scale and datum uncertainty will differ. The list of examples could go on and on because there are so many variables that affect the measurement uncertainty, and there are so many applications for the Electro-Limit Indicator.

To ensure that the list of uncertainty sources is comprehensive, it may help to organize the list chronologically by visualizing every step of the measurement process. The Electro-Limit Indicator is a calibrate-before-use instrument, so the measurement starts with setting the instrument's scale. The process of setting the scale will include setting two points – likely by resting the indicator on two different gage blocks – so the uncertainty of the gauge block's measurement, the parallelism of the gauge faces, and the resolution of those gage blocks will be considered as sources of uncertainty. One would also include the repeatability and resolution of the Electro-Limit Indicator as part of this step; both will be factors in establishing the two scale points. Furthermore, temperature variation of the measurement environment will affect the lengths of master gage blocks, so the expected variation due to your measurement environment should be included on the list of sources of uncertainty. After the measurement scale has been established, the user of the Electro-Limit Indicator must set a measurement datum. The datum point will also be subject to the same sources of uncertainty that affected the process of setting the measurement scale: resolution of the datum artifact, uncertainty of measurement of the datum artifact, variation due to geometry of the datum artifact, variation of the datum artifact due to environmental changes, and the resolution and repeatability of the Electro-Limit Indicator. Finally, after establishing the measurement scale and the measurement datum, the Electro-Limit Indicator is employed in measurement. The measurement event itself will include many of the same sources of uncertainty as the setting of the scale and the setting of the datum, so it may be appropriate to add another line for the resolution of the Electro-Limit Indicator, the repeatability of the Electro-Limit Indicator, the expected performance of the Electro-Limit Indicator (published as the accuracy of the instrument), and any instability due to vibration or temperature variation in the measurement environment. When visualizing this final step of the measurement process, one should also consider any uncertainty that may arise due to unique features of the measurement such as the contact geometry of the measurement, the compressibility of the unit under test, and the ability to orient the unit under test so that the proper dimension is being gauged.

[Estimate the Magnitude of the Effect of each Source of Uncertainty](#)

Once a comprehensive list of all of the uncertainty sources affecting all steps of the measurement process has been compiled, a value must be assigned to each source. This is the generation of an uncertainty budget.

[Find some values on the master gages' Certificates of Calibration](#)

For most of the Electro-Limit's measurement applications, gage blocks are used to set the instrument scale. In that case, several values arising from the gage blocks can be used to account for the uncertainty of the probe position when the two scale points are set. We should consider the resolution of the

reported measurement value, the uncertainty of the calibration of the gage blocks, and the parallelism of the gage block faces – all values that should be reported on the certificate of calibration for the gage blocks. Since the Electro-Limit Indicator utilizes point contact, the parallelism value need not be included on the uncertainty budget as long as the certificate specifies which points on the blocks' faces the measurement values refer to, and the user is careful to position gage blocks so that the Electro-Limit Indicator probe contacts the point that correlates to the reported value. The uncertainty of the gage blocks' calibration will likely be considerably larger than the resolution of the gage block, so it would be fair to include only the uncertainty value of the gage blocks' measurement on the uncertainty budget, but there may be ISO requirements demanding that resolution is included on the uncertainty budget.

Coverage Factor / Distribution Type

The values transferred from the master gage's certificate of calibration to your uncertainty budget must be processed before being entered to the uncertainty budget. The uncertainty of measurement of the masters will likely be expressed at a coverage factor of $k=2$. This means that the uncertainty value identifies the limits of a range of values where the authority issuing the measurement report is 95% certain that the true value of the measurement is within those limits. Generally, when making an uncertainty budget, the author will work with values expressed at $k=1$. So, if the uncertainty value found on the master gages' certificate of uncertainty is expressed at $k=2$, then the value should be halved before entering it to the uncertainty budget. Similarly, if the uncertainty budget author chooses to include the values for the master gauges' parallelism and resolution to the uncertainty budget, these values should also be reduced before entry. Let us consider this example for calculating the uncertainty contribution due to the resolution of the master gauge blocks: Let's say that the master gauge block certificate of calibration reports measurement values out to six places – x.xxxxxx inches or 10^{-6} inches. That means that, in terms of resolution alone, the length of the block could be 0.0000005 inches (5×10^{-7} inches) greater or lower than the reported value. For example, if a certificate reports the size of a gauge block as 0.100000, it is equally likely that the block length is 0.9999995, 0.100000, 0.1000004, 0.9999997, etc – any value within 0.0000005 inches of the reported value. This situation is called a rectangular distribution. Without getting into the equations behind it, when a range of possible values represented by a single value are distributed in a rectangular distribution, the appropriate uncertainty value to be entered on an uncertainty budget will equal the range of possibilities divided by $2 \times \text{square root of } 3$. If the certificate of calibration for the Electro-limit Indicator's master gage blocks reads out to six places, then the uncertainty due to the gages' resolution will be $1/(2 \times \sqrt{3}) = 2.89 \times 10^{-7}$ inches. If the certificate of calibration for the Electro-limit Indicator's master gage blocks reads out to seven places, then the uncertainty due to the gages' resolution will be $1/(2 \times \sqrt{3}) = 2.89 \times 10^{-8}$ inches.

Determine Master Gage Stability

In addition to these values that can be found on the certificate of calibration for the gage blocks, estimates for the stability of the gages in the user's measurement environment may be needed. These values can be found by multiplying the expected temperature range of the gage blocks in the measurement environment by the length of the gage block and then by the coefficient of thermal expansion of the gage block material. Likely, for the Electro-Limit Indicator, these values will be very small because the scale can be set with short blocks – say 0.100 to 0.110 inches. Normally the coefficient of thermal expansion for steel gage blocks is about 6.5 parts per million per °F. If a line item for vibration is included in the uncertainty budget, the value can be determined by running the GageCal 'total indicated reading' function for a few seconds.

All of the sources named thus far, and the ways to estimate their values, can be counted again for the establishment of the measurement datum. Of course, if a different gage block is used to set the measurement datum, then it will likely have a different uncertainty value and parallelism from that of the blocks used to set the instrument scale. If it is a longer block (the Electro-Limit Comparator has capacity for a 6 inch block), then its sensitivity to temperature variation may be more significant; $6 \text{ inches} * 6.5 \text{ ppm}/^{\circ}\text{F} * 0.1^{\circ}\text{F} = 3.9 \times 10^{-6} \text{ inches}$ (for steel). The author of the uncertainty budget will need to know what length of datum block will be used for the measurement, and will need to know the typical thermal stability of the measurement environment. Here it is important to note a few truths about the effects of absolute temperature and temperature variation on dimensional measurement. Ideally, the temperature of the measurement environment would be at 68°F with no fluctuation at all. Of course, the temperature will only be exactly 68°F for a split second at a time, and will always fluctuate by some amount. With a calibrate-before-use instrument like the Electro-Limit Indicator, an absolute temperature that deviates from the standard temperature is not a problem, as long as the material and temperature of the unit under test matches the material and temperature of the gages used to set the scale. Imagine that the UUT and the master gages are all 90°. If their CTE all match, then all of their growth will be proportional, and the scale set with the masters will be a '90° F scale' that will give accurate readings for items of the same material as the master gages that are also at 90° F. Now, this does not mean that temperature effects can be ignored – there will be temperature fluctuation so that the temperature of the unit under test might not match that of the master gages. If all artifacts are steel, and the temperature of the artifacts fluctuates through a range of 0.1°F, then there could be errors as large as 0.65 ppm*artifact length. Also, CTE of steel may vary from artifact to artifact. NIST's [Gage Block Handbook](#) notes that CTE can vary by as much as 0.28 ppm/°F from one batch of steel to another. This relates to another factor that should be considered, calculated, and incorporated into the uncertainty budget – if the material of the unit under test is different from that of the master gages, then the uncertainty budget should include a value to account for different coefficients of thermal expansion and different compressibility. Those calculations are beyond the scope of this paper, but are only necessary if the UUT and the master materials differ from each other. Alternatively, the Electro-limit's GageCal software has a Temperature and Contact Correction feature that compensates for measurements where the unit under test is not the same material as the master datum, or for measurements where the unit under test is not the same material as the master datum and the two artifacts are not at the same temperature. Keep this in mind – except for unusual cases where the environment is very unstable, where the CTE of the part to be measured is different from that of the master and the temperature is not at the standard 68°F, or where the unit under test is much softer or harder than the datum master, a 'stability' component of the uncertainty budget that accounts for the datum masters' variation with room temperature (CTE*expected temperature range of block*length of block/(2*sqrt3)) and a component that accounts for the instrument's sensitivity to ambient vibration (TIR reading/(2*sqrt3)) will suffice.

[The Electro-Limit Indicator / Electro-Limit Comparator's Contribution](#)

The Electro-Limit itself will add some uncertainty to the measurement. The Electro-Limit's contribution should be separated into several line items – as long as we are organizing the sources of uncertainty chronologically as they appear in the measurement process. The specifications for the Electro-Limit Comparator include three values that can be incorporated into the uncertainty budget: resolution, accuracy, and repeatability. The resolution of the instrument is 0.1 μ-inch (1×10^{-7} inch). Remember to divide this value by $2 * \sqrt{3}$ for a very small contribution of 2.89×10^{-8} inches (0.0289 μ-inch). The

accuracy of the instrument is posted as '2 μin in any 0.004" range'. This means that as long as the difference between the length of the unit under test and the length of the master gauge is less than 0.004-inch, then the uncertainty is 2 μ-inch. If the difference between the two artifacts is greater than 0.004-inch, then the uncertainty will increase at the rate of 2 μ-inch per 0.004-inch. The direct reading range of the Electro-Limit Indicator is 0.01-inch, so the largest possible uncertainty arising from the 'accuracy' value would be $(0.01/0.004)*2 \mu\text{-inch} = 5 \mu\text{-inch}$. Remember to divide the accuracy value by 2 before entering the value on the uncertainty budget; the accuracy is expressed at $k=2$, but we want $k=1$ for entry to our uncertainty budget. The value to enter to the uncertainty budget to account for the 'accuracy' contributor should be between 1 and 2.5 μ-inch. The published repeatability of the instrument is 1.6 μ-inch. This value represents a band of + and – 2 standard deviations, so before entering the value to an uncertainty budget, it should be divided by 4, $1.6/4 = 0.4 \mu\text{-inch}$. The author of the uncertainty budget will need to determine which of these values will be counted one, two, three, or four times; depending on the sequence of operations to complete the measurement, the three values may affect the instrument scale, the datum, and the actual measurement.

Combine Values by Root-Sum-Square

Spreadsheet Software like Microsoft Excel or Libre Office will help greatly with processing the uncertainty budget. All uncertainty sources can be listed in one column. Their estimated magnitudes can be listed in the adjacent column. A third column can be used to calculate the squares of all of the entered values – this will make calculating the R-S-S easier. It is suggested to group the sources according to where they appear in the measurement process: first list the sources that affect the scale, then those that affect the datum, and finally those that affect the measurement event. Calculate the scale factor uncertainty by finding the root-sum-square of all of the sources associated with setting the instrument scale (sum the squares that appear in the third column, and take the square root of the sum). Multiply this value by the length of the scale used – this number should be very small; the RSS value will be some number of micro-inches, and it will be multiplied by the scale length that cannot exceed 0.01-inch. The value for the uncertainty of scale should be less than 2×10^{-8} inches. This is less than 1% of the expected total uncertainty – ALL SCALE FACTOR CALCULATIONS CAN BE IGNORED – Really you need not even add these to your uncertainty budget. Next, calculate the root-sum-square of the remaining uncertainty values, those associated with the setting the datum and making the measurement. Add the datum-measurement R-S-S value to the scale uncertainty value to find the total uncertainty (As mentioned above, the scale uncertainty value should be so small that it can be ignored). FINALLY, MULTIPLY THIS VALUE BY 2 TO CALCULATE THE TOTAL UNCERTAINTY AT $K=2$. This is the value that should be reported with your measurement values in the form '±X μ-inches @ $k=2$, 95% C.L.'

Minimizing Measurement Uncertainty

The uncertainty budget for measurements made with the Electro-Limit Indicator actually has a pretty short list of contributors. The adjustments that can be made to reduce the uncertainty value include just the basics of contact based dimensional metrology: Use master gauges with low uncertainty. Ensure that all artifacts and operating surfaces are clean. Allow all artifacts and instruments to acclimate for a sufficient length of time so that all temperatures are the same and will not fluctuate during the measurement session. For more information on this important metrology topic please see: [Maximizing Accuracy in Micro-inch Measurement](#).

Application

The process described in this paper can be applied to any type of dimensional measurement. The model subject for this discussion is the Electro-Limit Indicator. The Electro-Limit Indicator was designed to replace traditional indicators with a digital instrument that has a greater dynamic range than any extant indicator. The Electro-Limit Indicator has a measurement range of 0.010-inches and a worst case accuracy of 0.000005-inches (5 micro-inches) resulting in a dynamic range of 2000. The Electro-Limit Indicator operates with Pratt & Whitney's GageCal software that reads out at a 0.0000001-inch (0.1 micro-inch) resolution, allows operation in a number of different modes, and conveniently feeds data into the GageCal spreadsheet or the software of the user's choice. The Electro-Limit Indicator is also offered in concert with a specially designed stand as the Electro-Limit Comparator. The Electro-Limit Indicator can be used with an array of anvil types offered by Pratt & Whitney including the corner cube anvil designed for measurement of spheres / balls, the three ball nest anvil designed for measurement of artifacts with flat gauging faces (gauge blocks), the serrated anvil, and the Live Anvil. The Live Anvil is an anvil with a built in measurement sensor. When coupled with the Electro-Limit Comparator, the Live Anvil allows the user to accurately track the position of two opposing points on an artifact.

Read about the Electro-Limit Indicator here:

https://www.prattandwhitney.com/products/Electrolimit_Indicator/

Read about the Electro-Limit Comparator here:

https://www.prattandwhitney.com/products/Electrolimit_Comparator/

Read about the Live Anvil here:

https://www.prattandwhitney.com/Customer-Content/www/CMS/files/Live_Anvil_specifications.pdf

Read about the Corner Cube Anvil here:

https://www.prattandwhitney.com/customer-content/www/CMS/files//optimizing_ball_diameter_measurement.pdf

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Pratt & Whitney is a leading manufacturer of precision dimensional metrology instruments. Product lines include: Supermicrometer®, LabMaster®, Labmicrometer®, Laseruler®, and Measuring Machines